3/24/17 "An eye for an eye only ends up making the whole world blind."-Gandhi

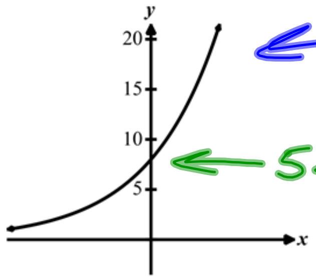
HW: "The Natural Exponent and the Natural Log" homework section
Test 3 on Thursday 3/30

AIM: What is the Natural Exponent?

Warm Up:

1. Given the graph of the exponential function $y = a(b)^x$ shown below, which of the following statements *must* be true?

(1) $a > 1$ and $b > 5$
(2) $a > 5$ and $b > 1$
(3) $a > 1$ and $b < 1$
(4) $a < 5$ and $b > 1$



increasing
$b > 1$

$5 < a < 10$

2. In the exponential function $g(x) = a(b)^x$ it is known that $g(3) = 25$ and $g(7) = 3$. Which of the following is closest to the value of b?

(1) 0.42
(3) 0.59
(2) 1.32
(4) 1.70

$(3, 25)$ $(7, 3)$

$$\frac{3 = \cancel{a}b^7}{25 = \cancel{a}b^3}$$

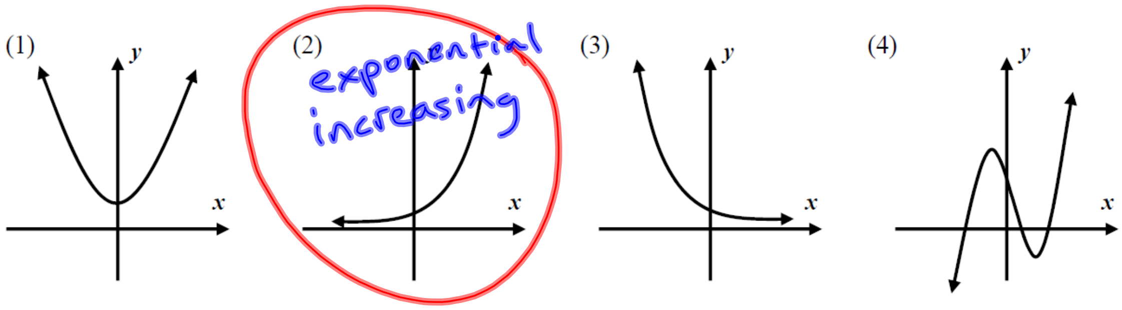
$$\frac{3}{25} = b^4$$

$$\sqrt[4]{\frac{3}{25}} = b$$

THE NUMBER e

1. Like π, e is irrational. 2. $e \approx 2.72$ 3. Used in Exponential Modeling

Exercise #1: Which of the graphs below shows $y = e^x$? Explain your choice. Check on your calculator.



Explanation: e is the base which is bigger than 1.

$y = ab^x$

Exercise #2: A population of llamas on a tropical island can be modeled by the equation $P = 500e^{0.035t}$, where t represents the number of years since the llamas were first introduced to the island.

(a) How many llamas were initially introduced at $t = 0$? Show the calculation that leads to your answer.

$P = 500e^{.035(0)}$

$P = 500e^0$

$P = 500(1)$

$\boxed{P = 500}$

(b) Algebraically determine the number of years for the population to reach 600. Round your answer to the nearest *tenth* of a year.

$P = 600$

$\frac{600}{500} = \frac{500e^{0.035t}}{500}$

$1.2 = e^{0.035t}$

$\frac{.035t}{.035} = \frac{\log_e 1.2}{.035}$

$t = \frac{\log_e 1.2}{.035}$

$\boxed{t \approx 5.2 \text{ years}}$

Because of the importance of $y = e^x$, its **inverse**, known as the **natural logarithm**, is also important.

THE NATURAL LOGARITHM

The inverse of $y = e^x$: $y = \ln x$ $(y = \log_e x)$

The natural logarithm, like all logarithms, gives an exponent as its output. In fact, it gives the power that we must raise **e** to in order to get the input.

Exercise #3: *Without* the use of your calculator, determine the values of each of the following.

(a) $\ln(e) = \boxed{1}$

what exponent do we raise e to to get e

(b) $\ln(1) = 0$

~~$\log_e(1)$~~

(c) $\ln(e^5) = 5$

(d) $\ln\sqrt{e}$

$\ln e^{1/2} = \frac{1}{2}$

Exercise #4: Which of the following is equivalent to $\ln\left(\frac{x^3}{e^2}\right)$?

(1) $\ln x + 6$

(2) $3\ln x - 2$

(3) $3\ln x - 6$

(4) $\ln x - 9$

$2\ln e$

$\ln x^3 - \ln e^2$

$3\ln x - 2$

Exercise #5: A hot liquid is cooling in a room whose temperature is constant. Its temperature can be modeled using the exponential function shown below. The temperature, T, is in degrees Fahrenheit and is a function of the number of minutes, m, it has been cooling.

$$T(m) = 101e^{-0.03m} + 67$$

(a) What was the initial temperature of the water at $m = 0$. Do without using your calculator.

$T(0) = 101e^{-.03(0)} + 67$

$= 101e^0 + 67$

$= 101(1) + 67$

$T(0) = 168$ degrees

(b) How do you interpret the statement that $T(60) = 83.7$?

In 60 minutes the temperature is 83.7° F

(c) Using the natural logarithm, determine algebraically when the temperature of the liquid will reach 100 °F. Show the steps in your solution. Round to the nearest tenth of a minute.

$100 = 101e^{-.03m} + 67$

$-67 \qquad\qquad -67$

$\frac{33}{101} = \frac{101e^{-.03m}}{101}$

$\frac{33}{101} = e^{-.03m}$

$\ln\left(\frac{33}{101}\right) = \ln e^{-.03m}$

$\ln\left(\frac{33}{101}\right) = -.03m\ln e$

$\frac{\ln\left(\frac{33}{101}\right)}{-.03} = \frac{-.03m(1)}{-.03}$

(d) On average, how many degrees are lost per minute over the interval $10 \le m \le 30$? Round to the nearest tenth of a degree.

When we solve for an EXPONENT we need logs.

$\ln e = 1$

When we are solving for an EXPONENT we need to use logs.

$$\log_e e^2 = \ln e^2$$

$$\ln x^3 = \log_e x^3$$